

# Vortex Cores, Circulation Cells, and Filaments in Geophysical Turbulence

The fluid dynamics of earth's atmosphere and oceans are strongly influenced by planetary rotation and vertical density stratification. The two-dimensional (2D) vorticity equation for barotropic turbulence and the three-dimensional quasi-geostrophic equation (3D QG) are standard models used to study the effects of rotation and stratification on large-scale turbulence.

The dominant behavior of freely decaying 2D turbulence is vortex formation and merger. Numerical experiments show that an initial distribution of random vorticity coalesces into a population of small vortices, which continue to merge with like-signed partners until the field is dominated by a few large vortices. Vortex merger is the mechanism for the inverse cascade, where energy moves from smaller to larger scales. This is in contrast to isotropic 3D turbulence, where vortex stretching and tilting transfers energy from large to small scales.

3D QG behaves like stacked layers of 2D turbulence which communicate through baroclinic interactions. Like the 2D vorticity equation, 3D QG lacks a stretching and tilting term, so energy travels up-scale in an inverse cascade by means of vortex merger. As time progresses, like-signed vortices in neighboring layers align to form vertical columns.

I am interested in studying the following questions:

1. How do components like vortex cores, circulation cells, and filaments each effect statistics such as the enstrophy spectrum and velocity probability density function (pdf)?
2. How does 3D QG differ from 2D turbulence in these measures, and what does this say about the validity of each equation to model geophysical turbulence?

The Okubo-Weiss parameter, defined as the strain squared minus the vorticity squared, is used to separate the vorticity field into separate components [3]. For 2D and 3D QG the Okubo-Weiss parameter is equivalent to the  $\lambda_2$  value, which is used to quantify and visualize coherent structures in fully 3D turbulence. Vortex cores, where the vorticity magnitude is large but the velocity is near zero, have large negative  $\lambda_2$  (Figure 3). The circulation cells are rings of high velocity induced by the vortex cores where  $\lambda_2$  is large and positive. These regions have lower vorticity than the cores and high shear strain due to the way velocity decreases with distance from the core. Background areas with  $\lambda_2$  near zero are neither vortex cores nor circulation cells, and include the low-

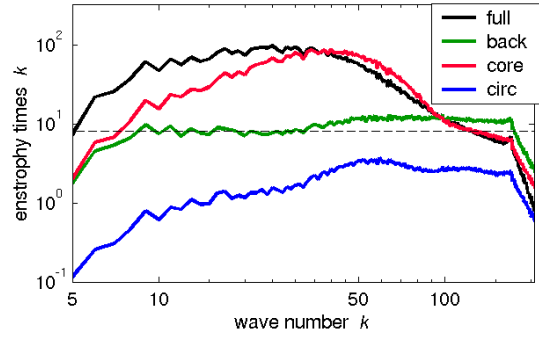


Figure 1: Enstrophy spectra of components of a 2D turbulence simulation. Isotropic 2D turbulence is proportional to  $k^{-1}$ , which is horizontal on these plots. The enstrophy spectra for circulation cells and the background are closer to  $k^{-1}$  spectra than the vortex core spectra.

vorticity filamentous structures that fill the majority of the vorticity field.

My current work studies decaying turbulence on the  $f$ -plane, where the Coriolis force does not vary across the domain. Numerical simulations were conducted at  $512^2$  in 2D and  $512^3$  in 3D QG using a pseudo-spectral model with periodic boundary conditions, hyperviscosity, and a viscosity coefficient of  $10^{-9}$ .

Vortex cores, circulation cells, and the remaining background have distinct effects in rotating turbulence. The background and circulation cells fit the  $k^{-1}$  enstrophy spectrum for isotropic turbulence, while the vortex cores do not (Figure 1). The vortex cores induce strong localized circulation that result in non-Gaussian velocity pdfs, as predicted by theoretical point vortex models. The background produces diffuse, nonlocalized velocity fields which have more Gaussian pdfs (Figure 2).

The study of velocity pdfs in turbulent flow is relevant to parameterizations of particle dispersion in ocean modeling. Most parameterizations by linear stochastic process or eddy-diffusivity assume Gaussian velocity distributions, so that the variance of velocity data can be used to estimate turbulent diffusivity. Physical data from ocean floats [2] and numerical models of the North Atlantic [1] both indicate that large-scale ocean circulation has non-Gaussian velocity pdfs with kurtosis values which are often greater than 5. Kurtosis values in that range only occur in our data due to the vortex cores and circulation

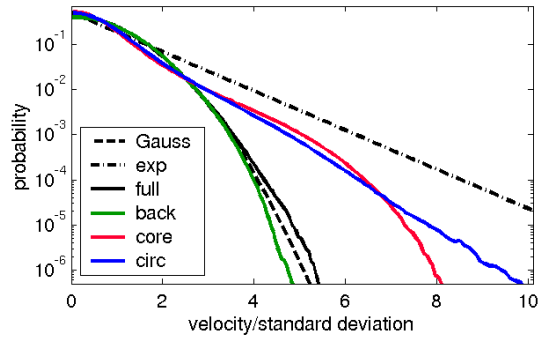


Figure 2: Velocity pdfs induced by components of a 3D QG simulation. The velocity due to the full field and the background are Gaussian, while circulation cells and the background are non-Gaussian.

cells. This suggests that vortices and coherent structures play a fundamental role in ocean circulation, and that diffusive parameterizations based solely on a Gaussian variance are inappropriate.

The filament-containing background plays a much stronger role in 3D QG than in 2D dynamics, as quantified by the energy and enstrophy in each field. The strong influence of the background results in nearly Gaussian velocity pdfs in 3D QG (kurtosis=3.08), but less Gaussian in 2D (kurtosis=3.95). This may indicate that 2D turbulence is a better model for ocean velocity distributions than 3D QG, because both 2D turbulence models and ocean observations produce non-Gaussian velocity pdfs.

## References

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- [2] Annalisa Bracco, J H LaCasce, and Antonello Provenzale. Velocity probability density functions for oceanic floats. *J Phys Oc*, 30:461–474, 2000.
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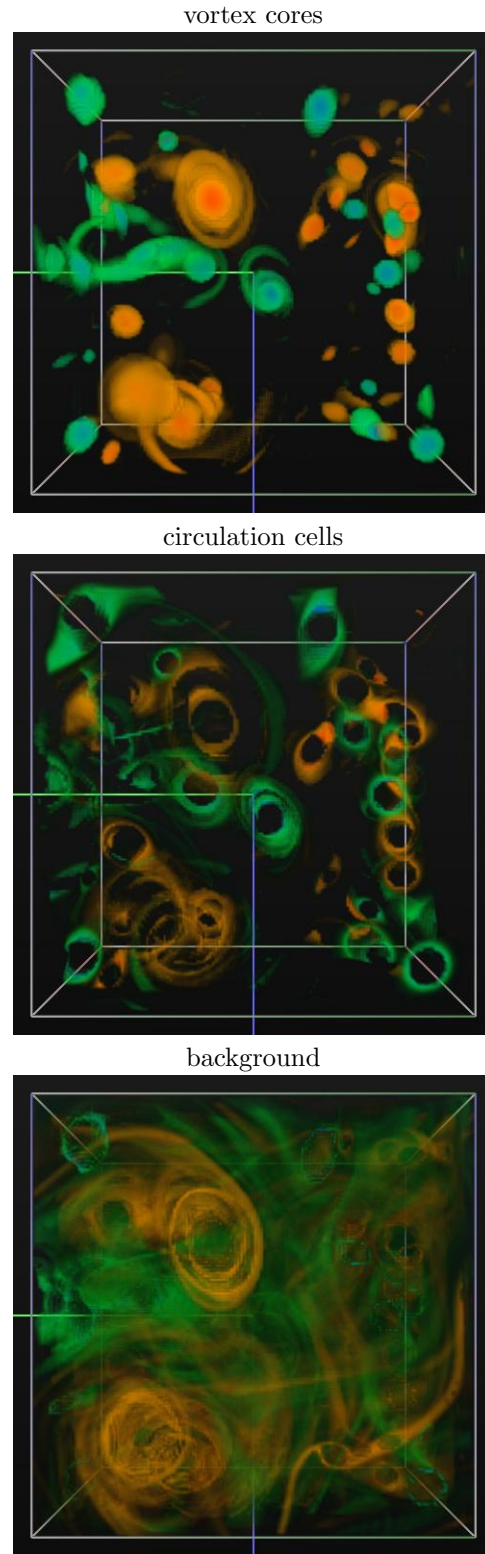


Figure 3: Volume rendered images of the vortex cores, circulation cells, and the filament-containing background in 3D QG, viewed from the top. Transparent (black) areas have potential vorticity near zero, while green (positive) and orange (negative) areas have larger potential vorticity magnitude.